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## Perturbative QCD Fragmentation Functions as a Model for Heavy-Quark Fragmentation

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### Abstract

The perturbative QCD fragmentation functions for a heavy quark to fragment into heavy-light mesons are studied in the heavy-quark limit. The fragmentation functions for S-wave pseudoscalar and vector mesons are calculated to next-to-leading order in the heavy-quark mass expansion using the methods of heavy-quark effective theory. The results agree with the  $m_b \rightarrow \infty$  limit of the perturbative QCD fragmentation functions for  $\bar{b}$  into  $B_c$  and  $B_c^*$ . We discuss the application of the perturbative QCD fragmentation functions as a model for the fragmentation of heavy quarks into heavy-light mesons. Using this model, we predict the fraction  $P_V$  of heavy-light mesons that

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are produced in the vector meson state as functions of the longitudinal momentum fraction  $z$  and the transverse momentum relative to the jet axis. The fraction  $P_V$  is predicted to vary from about  $1/2$  at small  $z$  to almost  $3/4$  near  $z = 1$ .

# I. Introduction

Heavy-quark spin-flavor symmetries are very useful for understanding the properties of hadrons containing a single heavy quark in kinematic regimes where nonperturbative aspects of the strong interaction are dominant. These symmetries arise from the fact that the charm, bottom, and top quarks are much heavier than  $\Lambda_{\text{QCD}}$ . The symmetry is exact in the limit of infinite quark mass, and corrections can be systematically organized into an expansion in powers of  $\Lambda_{\text{QCD}}/m_Q$  using heavy-quark effective theory (HQET). There has been much progress on the applications of heavy-quark symmetries and HQET to the spectroscopy, and to both exclusive and inclusive decays, of charm and bottom hadrons [1].

It has recently been pointed out by Jaffe and Randall [2] that HQET can also be applied to the fragmentation of a heavy quark into hadrons containing a single heavy quark. They showed that when the fragmentation function is expressed in terms of an appropriate scaling variable, it has a well-defined heavy-quark mass expansion. Specifically, they showed that the fragmentation function  $D_{Q \rightarrow H}(z)$  at the heavy-quark mass scale has a systematic expansion in inverse powers of  $m_Q$  when expressed as a function of the scaling variable

$$y = \frac{1 - (1 - r)z}{rz}, \quad (1)$$

where  $r = (m_H - m_Q)/m_H$ ,  $m_H$  is the mass of the heavy hadron, and  $z$  is its longitudinal momentum fraction relative to the fragmenting heavy quark. In the case of a heavy-light meson,  $r$  can be interpreted as the ratio of the constituent mass of the light quark to the meson mass. For the pseudoscalar meson  $P$  and vector meson  $V$  of the same S-wave multiplet ( $^1S_0, ^3S_1$ ), the fragmentation functions at the scale  $m_Q$  have heavy-quark mass expansions of the form

$$D_{Q \rightarrow P}(z) = \frac{a(y)}{r} + b(y) + \mathcal{O}(r), \quad (2)$$

$$D_{Q \rightarrow V}(z) = \frac{a^*(y)}{r} + b^*(y) + \mathcal{O}(r), \quad (3)$$

where  $a^*(y) = 3a(y)$ . By heavy-quark spin symmetry, the leading terms differ by a spin

factor of 3, while spin splittings first appear at next-to-leading order in the functions  $b(y)$  and  $b^*(y)$ .

It was also realized recently that the fragmentation functions for mesons containing a heavy quark and a heavy antiquark can be computed using perturbative quantum chromodynamics (PQCD) [3, 4, 5]. The fragmentation functions for a  $\bar{b}$  to split into the S-wave  $\bar{b}c$  mesons  $B_c$  and  $B_c^*$  were calculated to leading order in  $\alpha_s$  in Ref. [6]. These fragmentation functions have been used to predict the production rates of the  $B_c$  meson at LEP and at the Tevatron [7, 8]. The general analysis of Jaffe and Randall must certainly apply to perturbative QCD fragmentation functions in the limit where the mass of the heavier quark is taken to infinity. It was verified explicitly in Ref. [6] that the PQCD fragmentation functions  $D_{\bar{b} \rightarrow B_c}(z)$  and  $D_{\bar{b} \rightarrow B_c^*}(z)$  reduce to the forms (2) and (3) with  $r = m_c/(m_b + m_c)$  in the limit  $m_b \rightarrow \infty$ .

Since the PQCD fragmentation functions are consistent with heavy-quark symmetry, they can be used as models for the fragmentation of heavy quarks into heavy-light mesons. In this paper, we show how the leading and next-to-leading terms in the  $1/m_Q$  expansions can be calculated directly from HQET. We then discuss the use of the PQCD fragmentation functions as a phenomenological model for the fragmentation of charm and bottom quarks into heavy-light mesons. As an application of this model, we consider the fraction  $P_V$  of heavy-light mesons that are produced in the vector-meson state.

## II. PQCD Fragmentation Functions from HQET

The HQET Lagrangian, including the leading and the  $1/m_Q$  terms, is given by [1]

$$\mathcal{L} = \bar{h}_v \left\{ i v \cdot D + \frac{1}{2m_Q} \left( C_1 (iD)^2 - C_2 (v \cdot iD)^2 - \frac{C_3}{2} g_s \sigma^{\mu\nu} G_{\mu\nu} \right) \right\} h_v \quad (4)$$

where

$$\begin{aligned} C_1 &= 1, \\ C_2 &= 3 \left( \frac{\alpha_s(\mu)}{\alpha_s(m_Q)} \right)^{-8/(33-2n_f)} - 2, \\ C_3 &= \left( \frac{\alpha_s(\mu)}{\alpha_s(m_Q)} \right)^{-9/(33-2n_f)}. \end{aligned} \quad (5)$$

These coefficients are all equal to 1 at the heavy-quark mass scale  $\mu = m_Q$ . The term proportional to  $C_2$  can be omitted in calculating physical quantities, because it can be eliminated using a field redefinition involving the equation of motion  $(v \cdot D)h_v = 0$  from the leading term in the Lagrangian. Our method for calculating the fragmentation function involves a heavy quark which is off-shell by an amount at least of order  $m_Q m_q$ . To demonstrate that the  $C_2$  term can still be omitted in this case, we keep it in our calculation throughout, and show that it cancels between the vertex and propagator corrections.

In our calculation, we need the Feynman rules derived from the HQET Lagrangian for (i) the heavy-quark propagator, including  $1/m_Q$  corrections, (ii) the heavy-quark-gluon vertex, including  $1/m_Q$  corrections, and (iii) the propagator for the small component of the Dirac field of the heavy quark. This last Feynman rule is needed in our calculation because the fragmenting heavy-quark is off its mass shell. The Feynman rule for a heavy-quark propagator is

$$\frac{i}{v \cdot k + \frac{C_1}{2m_Q} k^2 - \frac{C_2}{2m_Q} (v \cdot k)^2} \frac{1 + \not{v}}{2}, \quad (6)$$

where  $k$  is the residual 4-momentum of the heavy quark. The  $QQg$  vertex is

$$-ig_s T^a \left( v^\mu + \frac{C_1}{2m_Q} (k_1 + k_2)^\mu - \frac{C_2}{2m_Q} v \cdot (k_1 + k_2) v^\mu + i \frac{C_3}{2m_Q} \sigma^{\mu\nu} q_\nu \right), \quad (7)$$

where  $k_1$  and  $k_2$  are the residual 4-momenta of the incoming and outgoing quarks and  $q = k_2 - k_1$  is the momentum of the gluon. The Feynman rule for the propagator of the small component of the Dirac field of the heavy-quark is

$$\frac{i}{v \cdot k} \frac{1 + \not{v}}{2} \left( \frac{1}{2m_Q} \sigma^{\mu\nu} \hat{q}_\nu \right) \frac{1 - \not{v}}{2}. \quad (8)$$

To calculate the fragmentation functions, we follow the method introduced in Refs. [3] and [5] and applied in Ref. [6] to the fragmentation processes  $\bar{b} \rightarrow B_c$  and  $\bar{b} \rightarrow B_c^*$ . We

denote the pseudoscalar and vector  $Q\bar{q}$  mesons by  $P$  and  $V$ , respectively. Here  $Q$  is the heavy quark, and  $\bar{q}$  is the light antiquark. We calculate the cross section for producing a  $Q\bar{q}$  meson plus a light quark  $q$  with total 4-momentum  $K^\mu$ , divide it by the cross section for producing an on-shell  $Q$  with the same 3-momentum  $\vec{K}$ , and take the limit  $K_0 \rightarrow \infty$ . The fragmentation function is

$$D(z) = \frac{1}{16\pi^2} \int ds \theta \left( s - \frac{M^2}{z} - \frac{m_q^2}{1-z} \right) \lim_{K_0 \rightarrow \infty} \frac{\sum |\mathcal{M}|^2}{\sum |\mathcal{M}_0|^2}, \quad (9)$$

where  $M = m_Q + m_q$  is the mass of the meson in the nonrelativistic approximation,  $s = K^2$ ,  $\mathcal{M}$  is the matrix element for producing  $P + q$  or  $V + q$ , and  $\mathcal{M}_0$  is the matrix element for producing an on-shell  $Q$ . The calculation can be greatly simplified by using the axial gauge with the gauge parameter  $n^\mu = (1, 0, 0, -1)$  in the frame where  $K^\mu = (K_0, 0, 0, \sqrt{K_0^2 - s})$ . In this gauge, we need only consider the production of the  $Q\bar{q}$  meson plus  $q$  through a virtual  $Q$  of momentum  $K^\mu$ . The part of the matrix element  $\mathcal{M}$  that involves production of the virtual  $Q$  can be treated as an unknown Dirac spinor  $\Gamma$ . In the limit  $K_0 \rightarrow \infty$ , the same spinor factor  $\Gamma$  appears in the matrix element  $\mathcal{M}_0 = \Gamma u(K)$  for an on-shell  $Q$ . The Feynman diagram for  $Q^* \rightarrow Q\bar{q} + q$  is shown in Fig. 1. The usual projection of the  $Q\bar{q}$  onto a nonrelativistic  $^1S_0$  bound state reduces in the heavy-quark limit to the Feynman rule

$$Q\bar{q} \rightarrow \frac{\delta^{ij}}{\sqrt{3}} \frac{R(0)\sqrt{M}}{\sqrt{4\pi}} \gamma^5 \frac{1 + \not{v}}{2}, \quad (10)$$

where  $R(0)$  is the radial wavefunction at the origin for the meson and  $v^\mu$  is its 4-velocity. For the  $^3S_1$  state, the projection is the same except that  $\gamma^5$  is replaced by  $\not{\epsilon}$ , where  $\epsilon^\mu$  is the polarization 4-vector for the vector meson  $V$ . The rest of the amplitude corresponding to Fig. 1 is obtained by using the ordinary QCD Feynman rules for the light quark spinor and the  $q\bar{q}g$  vertex and HQET Feynman rules for the heavy-quark propagator and the  $Q\bar{Q}g$  vertex.

The amplitude  $\mathcal{M}$  for producing the  $^1S_0$  state, including  $1/m_Q$  corrections in the heavy-quark propagator and vertex, is

$$i\mathcal{M} = -\frac{8\sqrt{\pi}\alpha_s R(0)}{3} \frac{M^2\sqrt{M}}{m_q} \frac{1}{(s - m_Q^2)^2} \frac{1}{1 + \frac{C_1 m_q}{m_Q} - \frac{C_2}{2m_Q} v \cdot k} \left( g_{\mu\nu} - \frac{n_\mu k_\nu + k_\mu n_\nu}{n \cdot k} \right)$$

$$\times \bar{u}(p')\gamma^\mu\gamma^5(1+\not{p})\left(v^\nu + \frac{C_1}{2m_Q}k^\nu - \frac{C_2}{2m_Q}(v\cdot k)v^\nu + \frac{C_3}{4m_Q}(\gamma^\nu\not{k} - \not{k}\gamma^\nu)\right)\frac{1+\not{p}}{2}\Gamma \quad (11)$$

where  $k = m_q v + p'$  is the momentum of the virtual gluon and also the residual momentum of the fragmenting heavy quark:  $K = m_Q v + k$ . Note that the term proportional to  $n_\nu$  in the numerator of the axial-gauge propagator for the gluon vanishes after contracting with the Dirac factor. For the vector meson state, the  $\gamma^5$  in the above equation is replaced by  $\not{n}$ .

We are interested only in the sum of the first two terms  $a(y)/r + b(y)$  in the heavy-quark mass expansion, where  $r = m_q/(m_q + m_Q)$ . We calculate separately the contributions to the fragmentation functions from the leading terms in the HQET Feynman rules, from the  $1/m_Q$  corrections from the propagator, and from the  $1/m_Q$  corrections from the vertex. For the following we will detail the derivation for the  $^1S_0$  state, but only quote the results for the  $^3S_1$  state.

We first derive the fragmentation function  $D_{Q\rightarrow P}(z)$  with the leading terms in the HQET propagator and vertex only. The amplitude reduces to

$$i\mathcal{M}_1 = \frac{8\sqrt{\pi}\alpha_s R(0)}{3} \frac{M^2\sqrt{M}}{m_q} \frac{1}{(s-m_Q^2)^2} \bar{u}(p')\left(1 + \frac{v\cdot k}{n\cdot k}\not{n}\right)\gamma^5(1+\not{p})\Gamma. \quad (12)$$

Squaring and summing over spins and colors of the light quark, we get

$$\begin{aligned} \sum |\mathcal{M}_1|^2 = & \frac{64\pi\alpha_s^2 |R(0)|^2}{9} \frac{M^5}{m_q^2} \text{Tr}(\Gamma\bar{\Gamma}(1+\not{p})) \left[ \frac{z(1-z)}{M^3(1-(1-r)z)^2(s-m_Q^2)^2} \right. \\ & \left. + \frac{-1+z+3rz}{M(1-(1-r)z)(s-m_Q^2)^3} - \frac{4rM}{(s-m_Q^2)^4} \right]. \end{aligned} \quad (13)$$

The corresponding amplitude-squared for producing an on-shell heavy quark is

$$\sum |\mathcal{M}_0|^2 = \frac{3M}{z} \text{Tr}(\Gamma\bar{\Gamma}(1+\not{p})). \quad (14)$$

Substituting  $|\mathcal{M}|^2$  and  $|\mathcal{M}_0|^2$  into (9) and integrating over  $s$ , we get

$$\begin{aligned} D_{Q\rightarrow P}(z) = & \frac{2\alpha_s(2m_q)^2 |R(0)|^2}{81\pi m_q^3} \frac{rz^3(1-z)^2}{(1-(1-r)z)^6} \\ & \times \left(3(1-(1-r)z)^2 - 8rz(1-z) + 12rz(1-(1-r)z)\right). \end{aligned} \quad (15)$$

Expressing this in terms of  $y$  using (1) and expanding to next-to-leading order in  $r$ , we have

$$D_{Q \rightarrow P}(z) = N \frac{(y-1)^2}{ry^6} (3y^2 + 4y + 8) - N \frac{(y-1)^3}{y^6} (3y^2 + 4y + 8) + \mathcal{O}(r), \quad (16)$$

where  $N = 2\alpha_s^2 |R(0)|^2 / (81\pi m_q^3)$ . Therefore, in terms of  $a(y)$  and  $b(y)$ , the leading term in the HQET Lagrangian contributes

$$a(y) = N \frac{(y-1)^2}{y^6} (3y^2 + 4y + 8), \quad (17)$$

$$b_1(y) = N \frac{(y-1)^2}{y^6} \left( -(y-1)(3y^2 + 4y + 8) \right). \quad (18)$$

The corresponding calculation for the  $^3S_1$  state gives  $a^*(y) = 3a(y)$  and  $b_1^*(y) = 3b_1(y)$ . These contributions to  $D_{Q \rightarrow P}(z)$  and  $D_{Q \rightarrow V}(z)$  differ by a spin factor of 3, as required by heavy-quark spin symmetry.

Next we calculate the contributions from  $1/m_Q$  corrections in the heavy-quark propagator and the heavy-quark vertex. Expanding out the  $1/m_Q$  correction to the propagator in (11) to first order, the correction to the amplitude is

$$\begin{aligned} i\mathcal{M}_2 &= \frac{8\sqrt{\pi}\alpha_s R(0)}{3} \frac{M^2 \sqrt{M}}{m_q} \frac{1}{(s - m_Q^2)^2} \left( -C_1 \frac{m_q}{m_Q} + \frac{C_2}{2m_Q} (m_q + v \cdot p') \right) \\ &\times \bar{u}(p') \left( 1 + \frac{v \cdot k}{n \cdot k} \not{n} \right) \gamma^5 (1 + \not{p}) \Gamma. \end{aligned} \quad (19)$$

Keeping the interference terms in  $|\mathcal{M}_1 + \mathcal{M}_2|^2$ , summing over spins and colors, and inserting into (9), we find a  $1/m_Q$  correction to  $D_{Q \rightarrow P}(z)$ . Expressing this in terms of  $y$ , we find that the contribution to  $b(y)$  is

$$b_2(y) = N \frac{(y-1)^2}{y^6} \left( (-2C_1 + C_2 y)(3y^2 + 4y + 8) \right). \quad (20)$$

A similar calculation for the  $^3S_1$  state gives  $b_2^*(y) = 3b_2(y)$ . The  $1/m_Q$  correction to the amplitude in (11) from the heavy-quark vertex is

$$\begin{aligned} i\mathcal{M}_3 &= -\frac{8\sqrt{\pi}\alpha_s |R(0)|}{3} \frac{M^2 \sqrt{M}}{m_q} \frac{1}{(s - m_Q^2)^2} \left( g_{\mu\nu} - \frac{n_\mu k_\nu}{n \cdot k} \right) \bar{u}(p') \gamma^\mu \gamma^5 (1 + \not{p}) \\ &\times \left( \frac{C_1}{2m_Q} k^\nu - \frac{C_2}{2m_Q} (v \cdot k) v^\nu + \frac{C_3}{4m_Q} (\gamma^\nu \not{k} - \not{k} \gamma^\nu) \right) \frac{1 + \not{p}}{2} \Gamma. \end{aligned} \quad (21)$$



Keeping the interference terms in  $|\mathcal{M}_1 + \mathcal{M}_3|^2$ , we obtain after some work the contribution to  $b(y)$  and to  $b^*(y)$  due to the  $1/m_Q$  vertex correction

$$b_3(y) = N \frac{(y-1)}{y^5} \left( -C_2(y-1)(3y^2 + 4y + 8) + 6C_1(y-1)(y+2) - 12C_3y \right), \quad (22)$$

$$b_3^*(y) = 3N \frac{(y-1)}{y^5} \left( -C_2(y-1)(3y^2 + 4y + 8) + 6C_1(y-1)(y+2) + 4C_3y \right). \quad (23)$$

In (11), the  $(1 + \not{p})/2$  factor adjacent to  $\Gamma$  projects onto the *large* component of the heavy-quark spinors produced by the source  $\Gamma$ . There is also a contribution of order  $1/m_Q$  from the *small* component of the heavy-quark spinors of the fragmenting  $Q$  quark [2]. The corresponding amplitude is given by

$$\begin{aligned} i\mathcal{M}_4 = & -\frac{8\sqrt{\pi}\alpha_s R(0)}{3} \frac{M^2\sqrt{M}}{m_q} \frac{1}{(s-m_Q^2)^2} \left( g_{\mu\nu} - \frac{n_\mu k_\nu}{n \cdot k} \right) \\ & \times \bar{u}(p') \gamma^\mu \gamma^5 (1 + \not{p}) \left( \frac{1}{4m_Q} (\gamma^\nu \not{k} - \not{k} \gamma^\nu) \right) \frac{(1 - \not{p})}{2} \Gamma. \end{aligned} \quad (24)$$

The contributions to  $b(y)$  and to  $b^*(y)$  from the interference term in  $|\mathcal{M}_1 + \mathcal{M}_4|^2$  are

$$b_4(y) = 2N \frac{y-1}{y^5} (3y^3 + 5y^2 + 2y - 4), \quad (25)$$

$$b_4^*(y) = 6N \frac{y-1}{y^5} (y^3 - y^2 + 2y - 4). \quad (26)$$

The complete expression for  $b(y)$  is obtained by adding (18), (20), (22), and (25). Thus the fragmentation function  $D_{Q \rightarrow P}(z)$  for the  $^1S_0$  state, to next-to-leading order in  $1/m_Q$ , is given by (2) with

$$a(y) = N \frac{(y-1)^2}{y^6} (3y^2 + 4y + 8), \quad (27)$$

$$b(y) = N \frac{y-1}{y^6} \left( (y-1)(3y^3 + 15y^2 + 8y - 8) - 12(C_3 - 1)y^2 \right). \quad (28)$$

The complete expression for  $b^*(y)$  is obtained by adding  $3b_1(y)$ ,  $3b_2(y)$ , (23), and (26). The fragmentation function  $D_{Q \rightarrow V}(z)$  for the  $^3S_1$  state, to next-to-leading order in  $1/m_Q$ , is given by (3) with

$$a^*(y) = 3N \frac{(y-1)^2}{y^6} (3y^2 + 4y + 8), \quad (29)$$

$$b^*(y) = 3N \frac{y-1}{y^6} \left( -(y-1)(y^3 + y^2 - 8y + 8) + 4(C_3 - 1)y^2 \right). \quad (30)$$

The terms proportional to  $C_2$  in (28) and (30) cancel between propagator and vertex corrections. We have set  $C_1 = 1$  in (28) and (30). If we further put  $C_3 = 1$ , we recover the next-to-leading terms in the  $1/r$  expansion of the PQCD fragmentation functions given in Ref. [6].

The heavy-quark mass expansions (2) and (3) break down in the limit  $y \rightarrow \infty$ , which corresponds to  $z \rightarrow 0$ , and also in the limit  $y \rightarrow 1$ , which corresponds to  $z \rightarrow 1$ . As  $y \rightarrow \infty$ , the leading terms, given by (27) and (29), scale like  $1/(ry^2)$ , while the next-to-leading terms in (28) and (30) scale like  $1/y$ . Thus the  $1/m_Q$  expansion breaks down when  $y$  is of order  $1/r$  or larger. In the limit  $y \rightarrow 1$ , the leading terms in (2) and (3) vanish like  $(y-1)^2/r$  while the terms proportional to  $C_3 - 1$  in the next-to-leading terms go to 0 as the first power of  $y - 1$ . Thus, unless  $C_3 = 1$ , the expansion also breaks down for  $y - 1$  of order  $r$  or smaller.

In Fig. 2, we compare the PQCD fragmentation functions (solid curves) with the heavy-quark mass expansions (2) and (3) at leading order (dotted curves) and next-to-leading order (dashed curves) in  $r$ . We use the value  $r = 0.10$ , which corresponds to  $D$  mesons. The normalization is fixed by arbitrarily setting  $N = 1$  in (27)-(30). Note that we have set  $C_3 = 1$  in (28) and (30). For any other value of  $C_3$ , either (2) or (3) becomes negative for  $y$  very close to 1 indicating the breakdown of the  $1/m_Q$  expansion when  $z$  is too close to 1. From the figure it is clear that the next-to-leading order curves are in very good agreement with the complete PQCD fragmentation functions for both  $D$  and  $D^*$  mesons. Surprisingly, the leading order result for fragmentation into  $D^*$  mesons also agrees very well with the complete PQCD result, while the leading order result for the  $D$  meson falls about 30% low near the peak.

### III. PQCD Model for Heavy-Quark Fragmentation

It is tempting to use the heavy-quark limits of the PQCD fragmentation functions as phenomenological models for the fragmentation of a heavy quark  $Q$  into heavy-light mesons  $Q\bar{q}$ , where  $Q = c$  or  $b$  and  $q = u, d$ , or  $s$ . To next-to-leading order in  $1/m_Q$ , these fragmenta-

tion functions are given by (2) and (3), with  $a(y)$ ,  $b(y)$ ,  $a^*(y)$ , and  $b^*(y)$  given in (27) – (30). In addition to  $N$  and  $r$ , we must treat  $C_3$  as a phenomenological parameter, since, according to (5), it depends on the low-energy scale  $\mu$  where perturbation theory breaks down. These 3 parameters all have well-defined scaling behavior with the heavy-quark mass; namely,  $N$  is independent of  $m_Q$ ,  $r$  scales like  $1/m_Q$ , and  $C_3$  scales like  $\alpha_s(m_Q)^{9/(33-2n_f)}$ . Thus, if the parameters are determined phenomenologically from data on charm fragmentation into  $D$  and  $D^*$  mesons, then the corresponding parameters for the  $B$  and  $B^*$  mesons can be determined by scaling. The problem with this model is that unless  $C_3 = 1$ , either  $D_{Q \rightarrow P}(z)$  or  $D_{Q \rightarrow V}(z)$  becomes negative for  $z$  near 1. This unphysical behavior only arises in a region of  $z$  where the  $1/m_Q$  expansion is breaking down, but it makes these fragmentation functions less attractive as a phenomenological model. If we choose  $C_3 = 1$  to avoid these difficulties, we might as well avoid the  $1/m_Q$  expansion altogether and use the complete PQCD fragmentation functions as our model. We therefore propose as a model of heavy quark fragmentation the PQCD fragmentation functions calculated in Ref. [6]:

$$D_{Q \rightarrow P}(z) = N \frac{rz(1-z)^2}{(1-(1-r)z)^6} \left[ 6 - 18(1-2r)z + (21 - 74r + 68r^2)z^2 - 2(1-r)(6 - 19r + 18r^2)z^3 + 3(1-r)^2(1-2r+2r^2)z^4 \right], \quad (31)$$

$$D_{Q \rightarrow V}(z) = 3N \frac{rz(1-z)^2}{(1-(1-r)z)^6} \left[ 2 - 2(3-2r)z + 3(3-2r+4r^2)z^2 - 2(1-r)(4-r+2r^2)z^3 + (1-r)^2(3-2r+2r^2)z^4 \right]. \quad (32)$$

The only parameters are the normalization  $N$ , which is independent of  $m_Q$ , and  $r$ , which scales like  $1/m_Q$ . The parameter  $r$ , which in the PQCD calculation has the value  $m_q/(m_Q + m_q)$ , can be interpreted as the ratio of the constituent mass of the light quark to the mass of the meson. Integrating over  $z$ , we obtain the total fragmentation probabilities:

$$\begin{aligned} \int_0^1 dz D_{Q \rightarrow P}(z) &= 3N \left( \frac{8 + 13r + 228r^2 - 212r^3 + 53r^4}{15(1-r)^5} + \frac{r(1 + 8r + r^2 - 6r^3 + 2r^4) \log(r)}{(1-r)^6} \right) \\ \int_0^1 dz D_{Q \rightarrow V}(z) &= 3N \left( \frac{24 + 109r - 126r^2 + 174r^3 + 89r^4}{15(1-r)^5} \right) \end{aligned} \quad (33)$$

$$+ \frac{\tau(7 - 4\tau + 3\tau^2 + 10\tau^3 + 2\tau^4) \log(\tau)}{(1 - \tau)^6} \Big) \quad (34)$$

The PQCD fragmentation functions (31) and (32) give the distributions in the longitudinal momentum fraction  $z$  for the mesons  $P$  and  $V$  in a heavy-quark jet. This model can be easily extended to give the distribution in their transverse momentum  $k_T$  relative to the jet momentum [21]. In Ref. [6], the fragmentation functions were obtained as integrals over the invariant mass  $s$  of the fragmenting heavy quark:

$$D_{Q \rightarrow P/V}(z) = \int_{s_{\min}(z)}^{\infty} \frac{ds}{s} d_{Q \rightarrow P/V}(z, s), \quad (35)$$

where the lower limit of the integration is:

$$s_{\min}(z) = \frac{M^2}{z} + \frac{\tau^2 M^2}{1 - z}. \quad (36)$$

The functions  $d_{Q \rightarrow P/V}(z, s)$  in the integrand are given by

$$\begin{aligned} d_{Q \rightarrow P}(z, s) = & 6NM^2\tau s \left[ \frac{(1-z)(1+\tau z)^2}{(1-(1-\tau)z)^2(s-(1-\tau)^2M^2)^2} \right. \\ & - \frac{[2(1-2\tau) - (3-4\tau+4\tau^2)z + (1-\tau)(1-2\tau)z^2]M^2}{(1-(1-\tau)z)(s-(1-\tau)^2M^2)^3} - \left. \frac{4\tau(1-\tau)M^4}{(s-(1-\tau)^2M^2)^4} \right] \end{aligned} \quad (37)$$

$$\begin{aligned} d_{Q \rightarrow V}(z, s) = & 6NM^2\tau s \left[ \frac{(1-z)(1+2\tau z + (2+\tau^2)z^2)}{(1-(1-\tau)z)^2(s-(1-\tau)^2M^2)^2} \right. \\ & - \frac{[2(1+2\tau) - (1+12\tau-4\tau^2)z - (1-\tau)(1+2\tau)z^2]M^2}{(1-(1-\tau)z)(s-(1-\tau)^2M^2)^3} - \left. \frac{12\tau(1-\tau)M^4}{(s-(1-\tau)^2M^2)^4} \right] \end{aligned} \quad (38)$$

The invariant mass  $s$  is related to  $k_T$  and  $z$  by

$$s = \frac{M^2 + k_T^2}{z} + \frac{m_q^2 + k_T^2}{1 - z}, \quad (39)$$

where  $M = m_Q + m_q$  in the nonrelativistic limit. If, instead of integrating over  $s$ , we integrate over  $z$  with  $k_T^2$  held fixed, we obtained the  $k_T$  distribution for the fragmentation process. Introducing the dimensionless variable  $t = k_T/M$ , we can define the  $k_T$ -dependent functions

$d_{Q \rightarrow P/V}(z, t)$  and  $D_{Q \rightarrow P/V}(t)$  by

$$\int_0^\infty dt D_{Q \rightarrow P/V}(t) = \int_0^\infty dt \int_0^1 dz d_{Q \rightarrow P/V}(z, t) \quad (40)$$

$$= \int_0^1 dz \int_{s_{\min}(z)}^\infty \frac{ds}{s} d_{Q \rightarrow P/V}(z, s). \quad (41)$$

This implies

$$D_{Q \rightarrow P/V}(t) = 2M^2 t \int_0^1 dz \frac{1}{z(1-z)s(z, t)} d_{Q \rightarrow P/V}(z, s(z, t)) \quad (42)$$

with

$$s(z, t) = M^2 \left( \frac{1+t^2}{z} + \frac{r^2+t^2}{1-z} \right)$$

Carrying out the integrals over  $z$ , we find

$$\begin{aligned} D_{Q \rightarrow P}(t) = & \frac{Nr}{2(1-r)^6} \frac{1}{t^6} \left\{ -24rt [4r^2 - (2+r+2r^2)t^2] \log(r) \right. \\ & - (1-r)t [30r^3 - r(61-20r+28r^2)t^2 - (3-48r+48r^2-12r^3)t^4] \\ & + 12t [4r^3 - r(2+r+2r^2)t^2 + (1-r)^2 t^6] \log\left(\frac{r^2+t^2}{1+t^2}\right) \\ & + 3 [10r^4 - 3r^2(11+2r+2r^2)t^2 + (3+4r+19r^2-6r^3)t^4 \\ & \left. + (3+12r-20r^2+8r^3)t^6] \operatorname{Arctan}\left(\frac{(1-r)t}{r+t^2}\right) \right\} \quad (43) \end{aligned}$$

$$\begin{aligned} D_{Q \rightarrow V}(t) = & \frac{3Nr}{2(1-r)^6} \frac{1}{t^6} \left\{ -8rt [12r^2 - (6+7r+2r^2)t^2] \log(r) \right. \\ & - (1-r)t [30r^3 - r(61+28r-20r^2)t^2 + (5-8r+8r^2+4r^3)t^4] \\ & + 4t [12r^3 - r(6+7r+2r^2)t^2 + (1-r)^2 t^6] \log\left(\frac{r^2+t^2}{1+t^2}\right) \\ & + [30r^4 - 3r^2(33+22r-10r^2)t^2 + (9+20r+r^2+22r^3+8r^4)t^4 \\ & \left. + (9-12r+4r^2+8r^3)t^6] \operatorname{Arctan}\left(\frac{(1-r)t}{r+t^2}\right) \right\}. \quad (44) \end{aligned}$$

In general, fragmentation functions  $D(z, \mu^2)$  depend not only on  $z$  but also on a factorization scale  $\mu$ . In a high energy process that produces a jet with transverse momentum  $p_T$ , the scale  $\mu$  should be chosen to be on the order of  $p_T$ . The functions (31) and (32) should be regarded as models for heavy-quark fragmentation functions at a scale  $\mu$  of order  $m_Q$ .

For values of  $\mu$  much larger than  $m_Q$ , the fragmentation functions (31) and (32) should be evolved from the scale  $m_Q$  to the scale  $\mu$  using the Altarelli-Parisi equation:

$$\mu^2 \frac{\partial}{\partial \mu^2} D_{Q \rightarrow H}(z, \mu^2) = \int_z^1 \frac{dy}{y} P_{Q \rightarrow Q}\left(\frac{z}{y}, \mu\right) D_{Q \rightarrow H}(y, \mu^2), \quad (45)$$

where  $P_{Q \rightarrow Q}(x)$  is the appropriate splitting function:

$$P_{Q \rightarrow Q}(x, \mu) = \frac{2\alpha_s(\mu)}{3\pi} \left( \frac{1+x^2}{1-x} \right)_+. \quad (46)$$

One aspect of the initial conditions (31) and (32) and the evolution equation (45) that may cause problems in practical applications is that they do not respect the phase space constraint:

$$D_{Q \rightarrow H}(z, \mu^2) = 0 \quad \text{for} \quad z < M^2/\mu^2, \quad (47)$$

This can be remedied [9] by using (47) as the initial condition on the fragmentation function equation and replacing (45) by the inhomogeneous evolution equation

$$\mu^2 \frac{\partial}{\partial \mu^2} D_{Q \rightarrow H}(z, \mu^2) = \int_z^1 \frac{dy}{y} P_{Q \rightarrow Q}\left(\frac{z}{y}, \mu\right) D_{Q \rightarrow H}(y, \mu^2) + d_{Q \rightarrow H}(z, \mu^2) \theta(\mu^2 - s_{\min}(z)), \quad (48)$$

where  $d_{Q \rightarrow H}(z, s)$  is defined by the integrand in (35) and  $s_{\min}(z)$  is given in (36).

## IV. Comparison with other Fragmentation Models

The model for heavy-quark fragmentation which has been used most extensively in phenomenological applications is the Peterson fragmentation function [10]:

$$D_{Q \rightarrow H}(z) = N_H \frac{z(1-z)^2}{[(1-z)^2 + \epsilon_H z]^2}, \quad (49)$$

where  $N_H$  and  $\epsilon_H$  are adjustable parameters that may depend on the hadron  $H$ . This fragmentation function has the correct behavior in the heavy-quark limit if  $N_H$  scales like  $1/m_Q$  and  $\epsilon_H$  scales like  $1/m_Q^2$ . Identifying  $\epsilon_H$  with  $\tau^2$  and expressing (49) in terms of the Jaffe-Randall scaling variable  $y$  defined in (1), we find that it reduces in the limit  $\tau \rightarrow 0$  to

$$D_{Q \rightarrow H}(z) \rightarrow \frac{N_H}{\tau^2} \frac{(y-1)^2}{[(y-1)^2 + 1]^2}. \quad (50)$$

The Peterson fragmentation function is just the square of a light-cone energy denominator multiplied by a phase space factor. It contains no spin information; the normalization parameter  $N_H$  is to be determined independently for the pseudoscalar and vector mesons of a heavy-quark spin multiplet.

An alternative fragmentation model which does contain spin information has been proposed by Suzuki [11]. Suzuki's fragmentation functions are derived from the same Feynman diagram in Fig. 1 as the PQCD fragmentation functions, but with two essential differences. First, the diagram was calculated in Feynman gauge. If a general covariant gauge had been used, Suzuki's fragmentation functions would have depended on the gauge parameter. The PQCD fragmentation functions that we calculated are gauge-invariant. We calculated the diagram in the axial gauge only for simplicity. If we had used a covariant gauge, we would have had to also include diagrams in which both the virtual heavy quark and the virtual gluon are emitted by the source  $\Gamma$  in Fig. 1. Alternatively, we could have calculated the PQCD fragmentation functions for the fragmentation of a heavy quark into S-wave heavy quarkonium directly from the general gauge-invariant definition [12]. Such a calculation has been carried out for the equal mass case of charmonium by Ma [13]. A second essential difference between the PQCD model and Suzuki's is that we integrated over the invariant mass  $s$  of the fragmenting quark (see Eq. (9)). The invariant mass is related to the transverse momentum  $k_T$  of the meson relative to the fragmenting quark by (39). Rather than integrating over  $k_T^2$ , Suzuki chose to evaluate the integrand at a typical value  $\langle k_T^2 \rangle$ . Suzuki's model therefore has 3 parameters: the overall normalization  $N$ , the mass ratio  $r$ , and  $\langle k_T^2 \rangle / m_Q^2$ . When expressed in terms of the scaling variable  $y$  defined in (1), Suzuki's fragmentation function  $D_{Q \rightarrow P}(z)$  reduces in the limit  $r \rightarrow 0$  to

$$D_{Q \rightarrow P}(z) \rightarrow \frac{N}{r} (y-1)^2 \frac{(y-2)^2 + \kappa^2}{[y^2 + \kappa^2]^4}, \quad (51)$$

where  $\kappa^2 = \langle k_T^2 \rangle / (r^2 m_Q^2)$ . By heavy-quark spin symmetry  $D_{Q \rightarrow V}(z)$  differs, in this limit, only by a factor of 3.

The Peterson, Suzuki, and PQCD fragmentation functions all vanish like  $(1-z)^2$  as  $z \rightarrow 1$ .

An alternative fragmentation function which vanishes like the first power of  $(1 - z)$  has been proposed by Collins and Spillers [14]. This was motivated by incorrect dimensional counting rules. The correct dimensional counting rules for QCD [15] do in fact give a limiting behavior of  $(1 - z)^2$  for the fragmentation function. The Collins-Spillers fragmentation function can be derived in a similar way to ours, except that in the Feynman diagram in Fig. 1, the interaction mediated by the virtual gluon is replaced by a point-like scalar Yukawa coupling between the meson, the heavy quark, and the light quark. Consequently, the denominator of the matrix element contains only one power of  $(s - m_Q^2)$ , in contrast to the 2 powers in (12). It is the omission of the gluon propagator that changes the behavior as  $z \rightarrow 1$  from  $(1 - z)^2$  to  $(1 - z)$ . Also, instead of integrating over the invariant mass of the fragmenting quark as in (9), Collins and Spillers, like Suzuki, evaluated the integral at a typical value  $\langle k_T^2 \rangle$ . Taking the scaling limit  $r \rightarrow 0$ , the fragmentation function of Collins and Spillers reduces to

$$D_{Q \rightarrow P}(z) \rightarrow \frac{2N}{r} (y - 1) \frac{(y - 2)^2 + \kappa^2}{[y^2 + \kappa^2]^2}, \quad (52)$$

where  $\kappa^2 = \langle k_T^2 \rangle / (r^2 m_Q^2)$ .

The various fragmentation models in the literature have been summarized in Ref. [16] and compared with experimental data on  $D$  and  $D^*$  production. The string models and parton cluster models are very different in spirit from those discussed above. One can derive analytic expressions for the heavy-quark fragmentation functions from the string models [17]. They contain a tunneling factor  $\exp(-Bm_H^2/z)$ , which suppresses the small- $z$  region. In the scaling limit, the Lund symmetric fragmentation function behaves like

$$D_{Q \rightarrow H}(z) \rightarrow N r^\beta e^{-B(m_H^2 + \langle k_T^2 \rangle)} (y - 1)^\beta. \quad (53)$$

Unless  $N$  scales like  $e^{Bm_Q^2} m_Q^{\beta+1}$ , this is inconsistent with heavy-quark symmetry, which requires the leading term to scale like  $m_Q$  as  $m_Q \rightarrow \infty$ .

The PQCD model for heavy-quark fragmentation has a number of advantages over those described above. First, it is rigorously correct in the limit  $m_q \gg \Lambda_{\text{QCD}}$ . Higher order perturbative corrections can be systematically calculated. Relativistic corrections can also



be calculated in terms of additional nonperturbative matrix elements [18]. Second, our model is consistent with heavy-quark symmetry in the limit  $m_Q \rightarrow \infty$ . The logarithms of  $m_Q$  that are predicted by HQET would be reproduced by the higher order perturbative corrections. The PQCD model is also more predictive than those in Refs. [10, 11, 14]. It describes spin-dependent effects, like Suzuki's model, but without introducing any additional parameters. The PQCD model not only predicts the  $z$ -dependence of the fragmentation functions but also their dependence on  $k_T$ , the transverse momentum of the meson relative to the jet. The fragmentation functions (31) and (32) apply only to S-wave mesons, but the fragmentation functions for higher orbital-angular-momentum states can also be calculated. The PQCD fragmentation functions for the P-wave mesons have been calculated to leading order in  $\alpha_s$  in Refs. [19].

## V. The Vector-to-Pseudoscalar Ratio

In any production process for heavy-light mesons, one of the most fundamental experimental observables is the ratio

$$P_V = \frac{V}{V + P}, \quad (54)$$

which measures the relative number of vector mesons  $V$  and pseudoscalar mesons  $P$  that are produced. If the mesons are produced within a heavy-quark jet, then  $V$  and  $P$  in (54) can be identified as the fragmentation probabilities for the heavy quark to fragment into vector and pseudoscalar mesons, respectively. The ratio  $P_V$  can depend on kinematic variables, such as the longitudinal momentum fraction  $z$  of the meson or its transverse momentum  $k_T$  relative to the jet. In the PQCD model for fragmentation, the normalization factor  $N$  cancels out in the ratio (54), so that  $P_V$  is determined by the parameter  $r$  only.

The simplest measure of the ratio  $P_V$  comes from the total numbers of vector and pseudoscalar mesons in the jet integrated over  $z$  and  $k_T$ . Setting  $P$  and  $V$  in (54) to the fragmentation probabilities in (33) and (34), we find that the ratio  $P_V$  in the PQCD model of

fragmentation is

$$P_V = \frac{(1-r)(24 + 109r - 126r^2 + 174r^3 + 89r^4) + 15r(7 - 4r + 3r^2 + 10r^3 + 2r^4)\log(r)}{2(1-r)(16 + 61r + 51r^2 - 19r^3 + 71r^4) + 60r(2 + r + r^2 + r^3 + r^4)\log(r)} \quad (55)$$

This ratio is plotted as a function of  $r$  in Fig. 3. From the graph it is clear that  $P_V$  is not strongly dependent on  $r$ . At  $r = 0$ ,  $P_V = 3/4$  as required by heavy-quark spin symmetry. As  $r$  increases  $P_V$  decreases slowly to  $P_V = 0.51$  at  $r = 0.5$ . Thus at nonzero values of  $r$ , the vector state is less populated than would be given by naive spin counting. We can determine the value of  $r$  for the  $D$  and  $D^*$  system using experimental measurements of  $P_V$ . A complete compilation of experimental data for  $P_V$  from LEP, CLEO, ARGUS, PETRA, and TRISTAN can be found in Ref. [20]. The key point in obtaining consistency between these measurements is using the updated branching ratio  $B(D^{**} \rightarrow D^0 \pi^+) \approx 0.68$  instead of the old value 0.55. The experimental value  $P_V = 0.65 \pm 0.06$  determines the parameter  $r_D$  for the  $D - D^*$  system to be  $r_D = 0.10^{+0.12}_{-0.07}$ . If we interpret  $r$  as the ratio of the constituent mass of the light quark to the mass of the meson, then the value  $r_D = 0.10$  corresponds to a constituent mass of 200 MeV. Given a value of  $r_D$ , we can determine the corresponding value for the  $B - B^*$  system by using the simple scaling behavior  $r_B = (m_D/m_B)r_D$ . This gives  $r_B = 0.03^{+0.04}_{-0.02}$ .

Having determined the parameter  $r$  from data on  $D - D^*$  production, we can now predict how the vector-to-pseudoscalar ratio should vary as a function of the longitudinal momentum fraction  $z$ . The  $z$ -dependent ratio  $P_V(z)$  is defined by (54), where  $P$  and  $V$  are given by the fragmentation functions (31) and (32):

$$P_V(z) = \frac{3}{4} \frac{n(z)}{d(z)}, \quad (56)$$

with

$$\begin{aligned} n(z) &= 2 - 2(3 - 2r)z + 3(3 - 2r + 4r^2)z^2 \\ &\quad - 2(1 - r)(4 - r + 2r^2)z^3 + (1 - r)^2(3 - 2r + 2r^2)z^4, \\ d(z) &= 3 - 3(3 - 4r)z + (12 - 23r + 26r^2)z^2 \end{aligned} \quad (57)$$

$$-(1-r)(9-11r+12r^2)z^3 + 3(1-r)^2(1-r+r^2)z^4. \quad (58)$$

This ratio is plotted as a function of  $z$  in Fig. 4 for the values  $r = 0.10$  (solid curve),  $r = 0.03$  (dotted curve), and  $r = 0.22$  (dashed curve). At  $z = 0$ ,  $P_V(z) = 1/2$ , regardless of the value of  $r$ . It decreases slightly for small  $z$ , and then increases monotonically to a maximum value, at  $z = 1$ , of 0.74 for  $r = 0.03$ , 0.73 for  $r = 0.10$ , and 0.70 for  $r = 0.22$ . Note that, in spite of the large uncertainty in our determination of  $r$ , the uncertainty in  $P_V(z)$  is less than about 11%. Thus the PQCD model gives a rather unambiguous prediction that  $P_V$  should vary from around 1/2 at small values of  $z$  to almost 3/4 near  $z = 1$ .

The  $k_T$ -dependent ratio  $P_V(k_T)$  is defined by (54), with  $P$  and  $V$  given by (43) and (44):

$$P_V(k_T) = \frac{3}{4} \frac{n_1 + n_2 \log(r) + n_3 \log\left(\frac{r^2+t^2}{1+t^2}\right) + n_4 \text{Arctan}\left(\frac{(1-r)t}{r+t^2}\right)}{d_1 + d_2 \log(r) + d_3 \log\left(\frac{r^2+t^2}{1+t^2}\right) + d_4 \text{Arctan}\left(\frac{(1-r)t}{r+t^2}\right)}, \quad (59)$$

where

$$n_1 = -(1-r)t[30r^3 - r(61 + 28r - 20r^2)t^2 + (5 - 8r + 8r^2 + 4r^3)t^4], \quad (60)$$

$$n_2 = -8rt[12r^2 - (6 + 7r + 2r^2)t^2], \quad (61)$$

$$n_3 = 4t[12r^3 - r(6 + 7r + 2r^2)t^2 + (1-r)^2t^6], \quad (62)$$

$$n_4 = [30r^4 - 3r^2(33 + 22r - 10r^2)t^2 + (9 + 20r + r^2 + 22r^3 + 8r^4)t^4 + (9 - 12r + 4r^2 + 8r^3)t^6], \quad (63)$$

and

$$d_1 = -(1-r)t[30r^3 - r(61 + 16r - 8r^2)t^2 + 3(1 + 2r - 2r^2 + 2r^3)t^4], \quad (64)$$

$$d_2 = -12rt[8r^2 - 2(2 + 2r + r^2)t^2], \quad (65)$$

$$d_3 = 6t[8r^3 - 2r(2 + 2r + r^2)t^2 + (1-r)^2t^6], \quad (66)$$

$$d_4 = 30r^4 - 9r^2(11 + 6r - 2r^2)t^2 + 3(1+r)^2(3 + 2r^2)t^4 + 3(3 - 4r^2 + 4r^3)t^6. \quad (67)$$

This ratio is plotted as a function of  $t = k_T/M$  in Fig. 5 for the three values  $r = 0.10, 0.03$ , and 0.22. At  $t = 0$ ,  $P_V(t) = 3/4$ , regardless of the value of  $r$ . As  $t$  increases,  $P_V(t)$  quickly

decreases to its asymptotic value at  $t = \infty$ . At  $t = 1$ ,  $P_V(t)$  is within 0.1% of its asymptotic value of 0.65 for  $r = 0.03$ , 0.62 for  $r = 0.10$ , and 0.60 for  $r = 0.22$ . Again we find that, in spite of the large uncertainty in  $r$ , we obtain a rather precise prediction for  $P_V$  as a function of  $k_T$ .

The PQCD fragmentation functions for vector mesons have been applied previously [21] as a phenomenological model to describe the fragmentation processes  $c \rightarrow D^*$  and  $b \rightarrow B^*$ . The fragmentation functions were separated into the transverse and longitudinal polarization components. The spin alignment, which measures the ratio of transverse to longitudinal polarizations, was calculated as a function of  $z$  and as a function of  $k_T$ . In the case of production of  $D^*$  by charm fragmentation, the spin alignment predicted by the PQCD fragmentation model was shown to be consistent with CLEO measurements [21]. In addition, the predicted value of the average longitudinal momentum fraction  $\langle z \rangle$  for  $c \rightarrow D^*$  and for  $b \rightarrow B^*$  was shown to be in excellent agreement with data from LEP, CLEO, and ARGUS [21]. The values of  $r$  used for  $D^*$  and  $B^*$  mesons in these comparisons were  $r = 0.17$  and  $r = 0.058$ , respectively, which lie within the range determined above from measurements of  $P_V$ .

The PQCD fragmentation functions have also been applied in Ref. [22] to predict the fragmentation spectra for the  $B_s$  and  $B_s^*$  mesons based on the production rates of the  $B_s$  mesons measured at LEP. Instead of treating the normalization  $N$  as a phenomenological parameter as advocated in this paper, the authors calculated  $N$  using the PQCD expression, which involves  $\alpha_s$  at the scale of the strange quark mass.

## VI. Summary

We have studied the heavy-quark mass limit of the PQCD fragmentation functions for producing S-wave mesons. The leading and next-to-leading terms in  $1/m_Q$  were calculated directly from HQET. The PQCD fragmentation functions were proposed as a phenomenological model for fragmentation into heavy-light mesons. With only 2 parameters, this model describes fragmentation into the  $^1S_0$  pseudoscalar meson state and the transverse and longi-

tudinal polarization states of the  $^3S_1$  vector meson. It describes not only the  $z$ -dependence of the fragmentation probabilities, but also their dependence on the transverse momentum  $k_T$  of the meson relative to the jet within which it is produced. This model can easily be extended to describe heavy quark fragmentation into  $P$ -wave states using the PQCD fragmentation functions calculated in [19]. The PQCD fragmentation functions were compared with other models for heavy-quark fragmentation in the literature. As an application, the PQCD fragmentation functions were used to predict the ratio of vector-to-pseudoscalar states as a function of  $z$  and as a function of  $k_T$ . The ratio  $P_V$  is predicted to vary from around  $1/2$  at small values of  $z$  to almost  $3/4$  near  $z = 1$ .

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## Figure Captions

1. Feynman diagram used to calculate the PQCD fragmentation functions in axial gauge.
2. Comparison of the  $D_{c \rightarrow D}(z)$  (lower set of curves) and  $D_{c \rightarrow D^*}(z)$  (upper set of curves) fragmentation functions. The normalization is arbitrary. Shown are the full PQCD results (solid curves), the leading terms (dotted curves) in the heavy-quark mass expansion, and the leading plus next-to-leading terms (dashed curves) in the heavy-quark mass expansion.
3. The ratio  $P_V$  as a function of  $r$ .
4. Predictions for the ratio  $P_V(z)$  as a function of  $z$  for  $r = 0.10$  (solid curve),  $r = 0.03$  (dotted curve), and  $r = 0.22$  (dashed curve).
5. Predictions for the ratio  $P_V(k_T)$  as a function of  $k_T$  for  $r = 0.10$  (solid curve),  $r = 0.03$  (dotted curve), and  $r = 0.22$  (dashed curve).



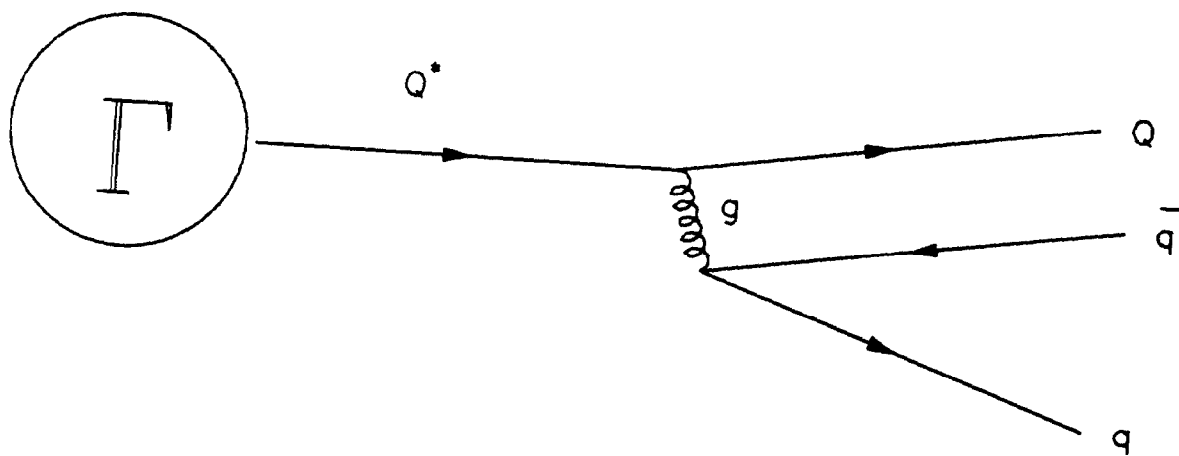


Figure 1

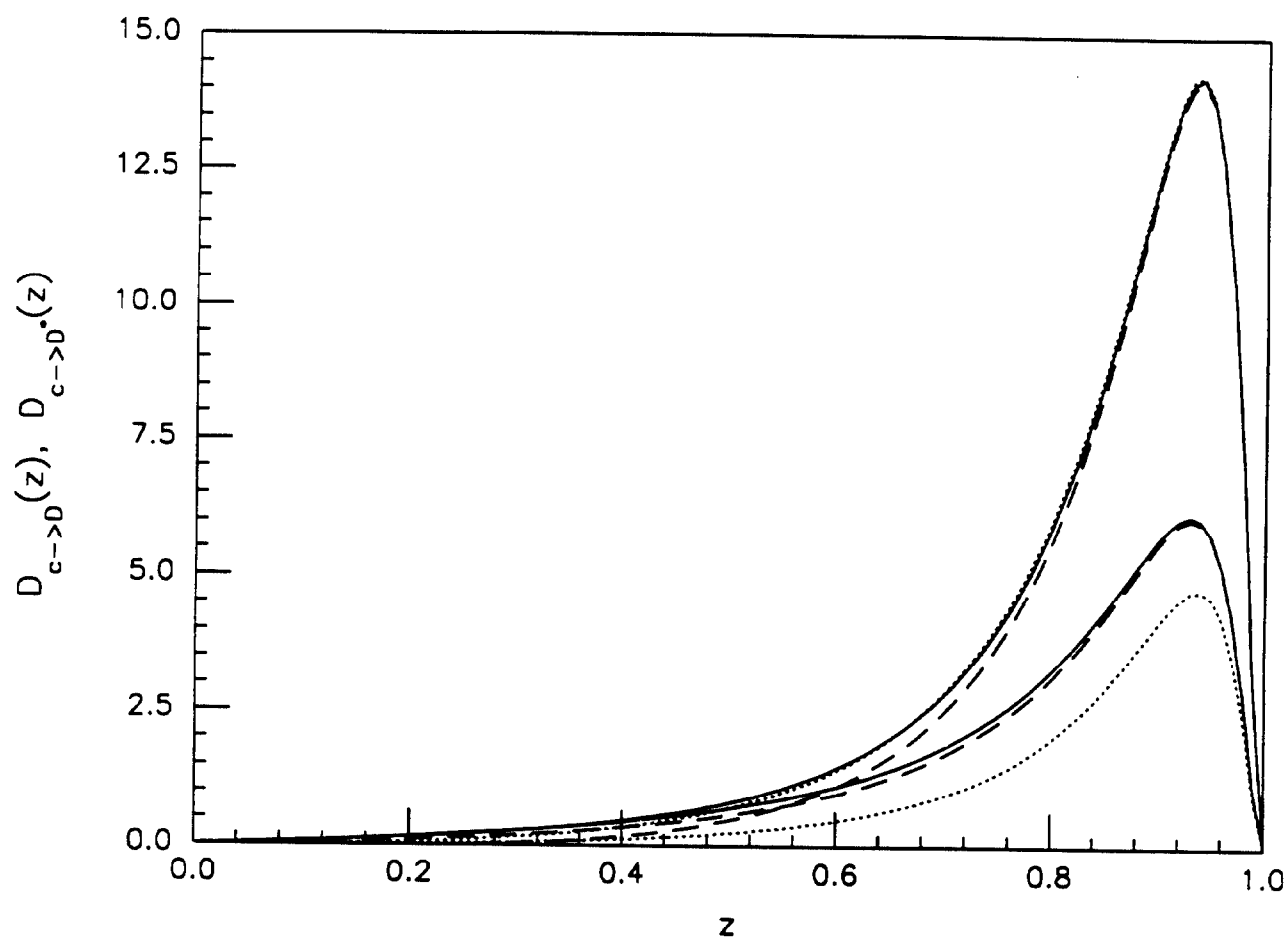


Figure 2

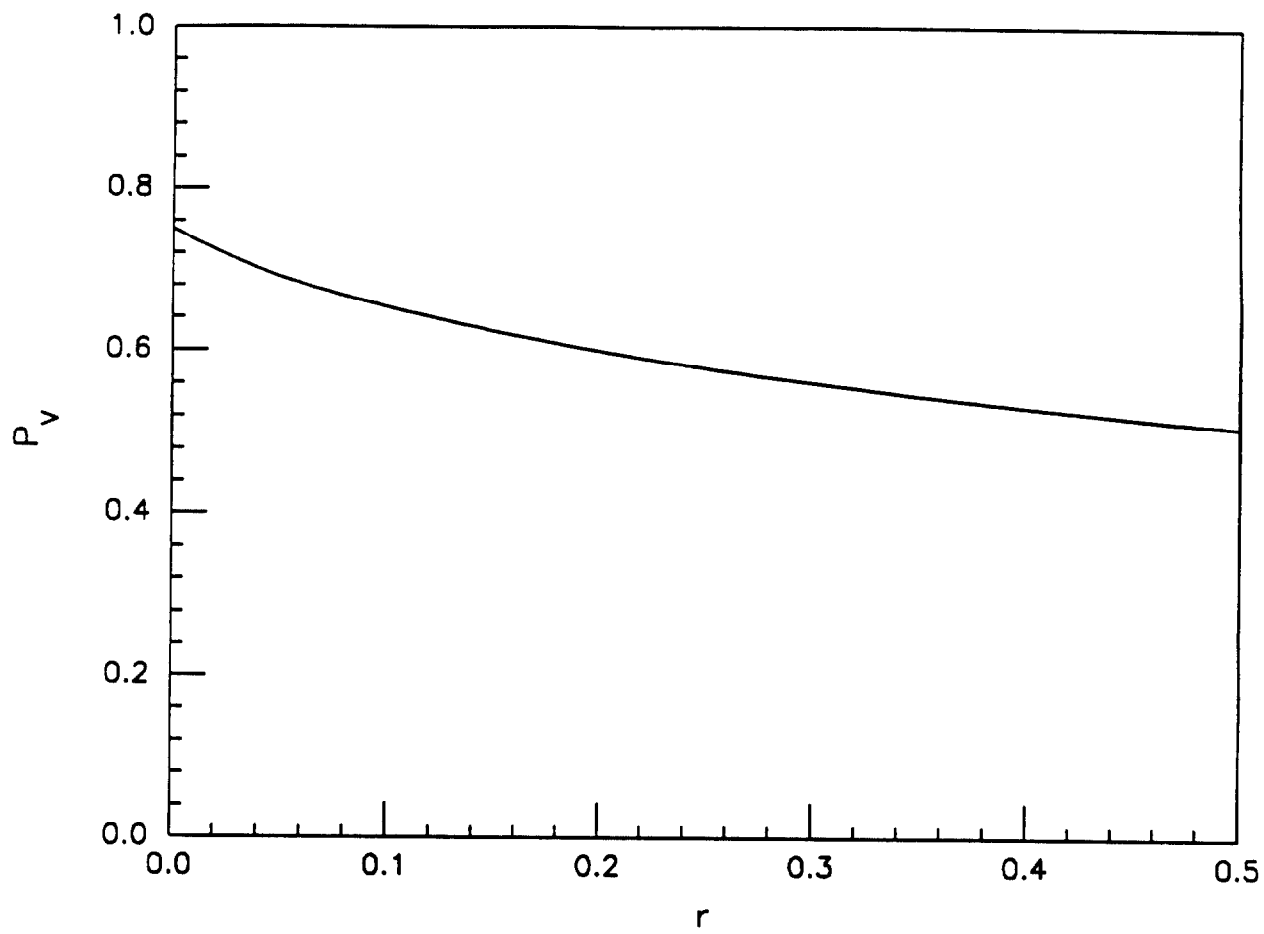


Figure 3

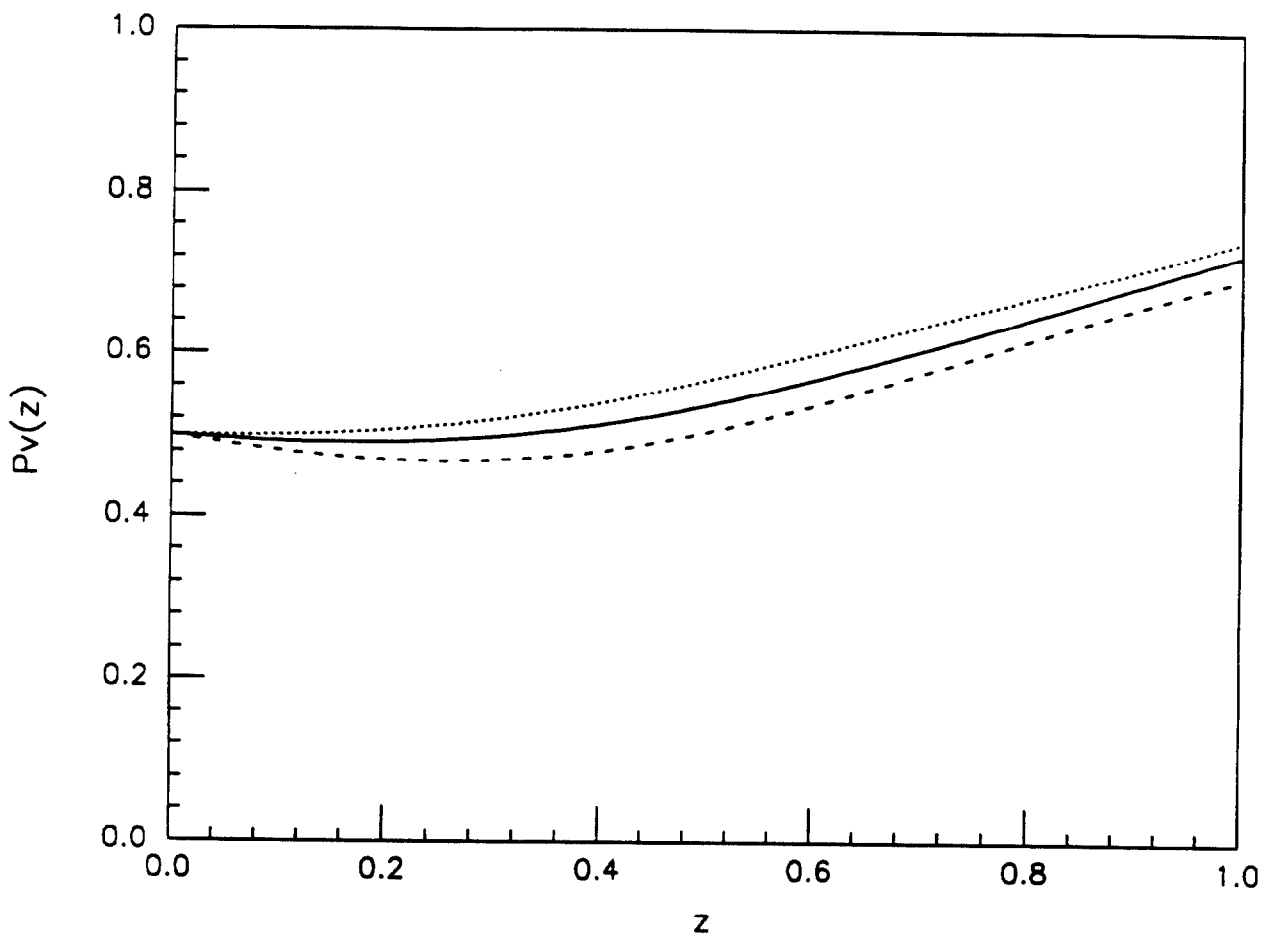


Figure 4

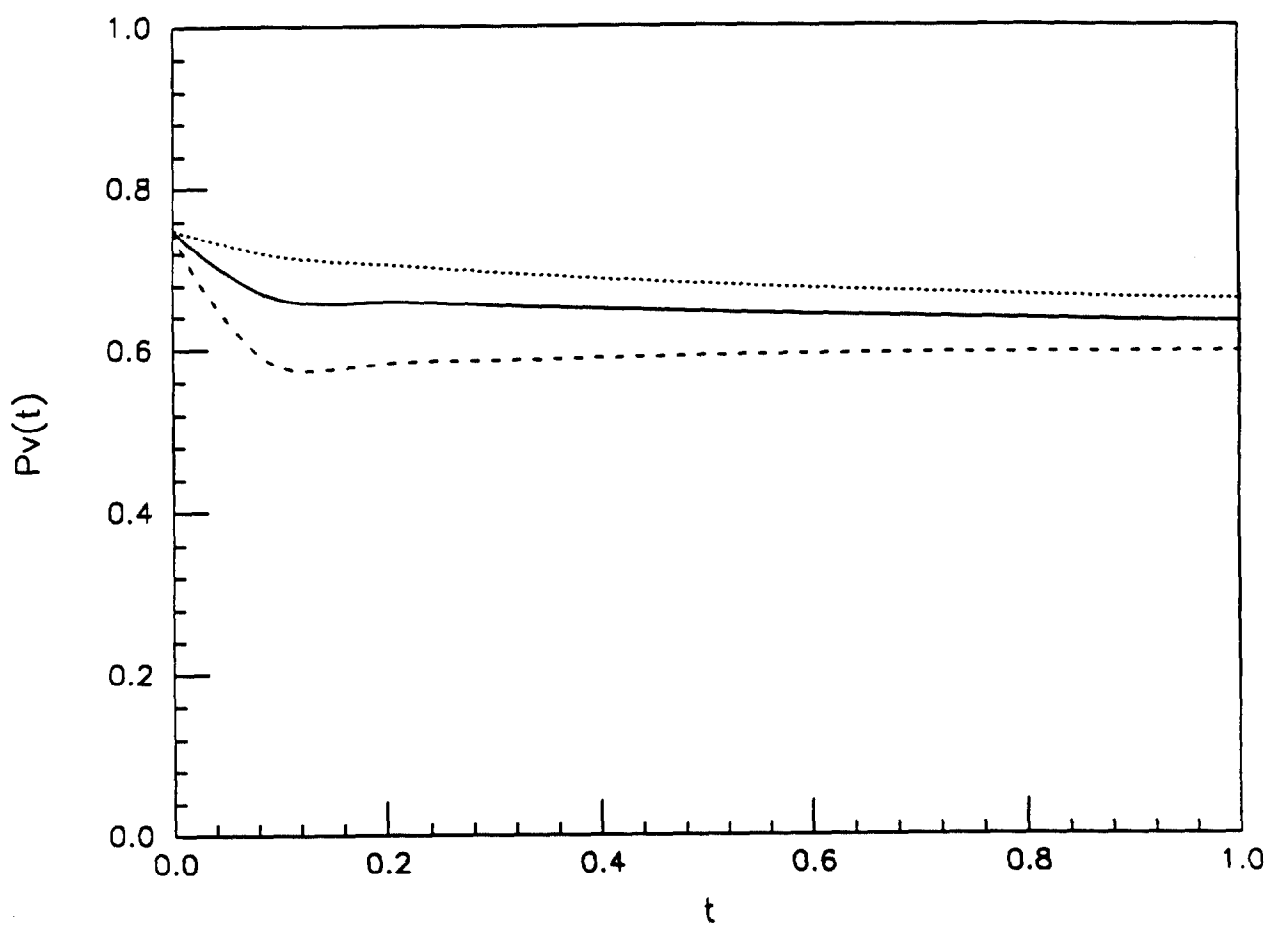


Figure 5